The Diffusion of Inflation Expectations: Theory and Evidence*

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Abstract: The assumption of homogenous representative agents with perfect rationality is challenged by recent macroeconomic literature. In this paper, we consider an asymmetric information diffusion process from more-informed to less-informed agents in a standard cobweb-type expectation model. Our main finding is that less-informed agents' forecasts confound those of more-informed agents whenever there is misinterpretation in the information acquisition process. We term this situation the boomerang effect. Using inflation forecasting data from Survey Research Center, we find the boomerang effect exists in agents' inflation forecasting behavior. The implication of the boomerang effect centers on policymaker's disseminating policy/public information in a transparent manner.

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Abstract

The assumption of homogenous representative agents with perfect rationality is challenged by recent macroeconomic literature. In this paper, we consider an asymmetric information diffusion process from more-informed to less-informed agents in a standard cobweb-type expectation model. Our main finding is that less-informed agents' forecasts confound those of more-informed agents whenever there is misinterpretation in the information acquisition process. We term this situation the boomerang effect. Using inflation forecasting data from Survey Research Center, we find the boomerang effect exists in agents' inflation forecasting behavior. The implication of the boomerang effect centers on policymaker's disseminating policy/public information in a transparent manner.

1. Introduction

Information diffusion and information asymmetries have both been documented in many areas of research. For example, financial economists have studied explanations for herding behavior, in which rational investors demonstrate some degree of behavioral convergence (see Devenow and Welch, 1996).[1] Most recently studies of monetary economics are exploring how information diffusion influences economic forecasting behavior. The standard monetary view from the "credibility" literature holds that policymakers have superior information to citizens and hence can choose how much information to disseminate for better stabilization outcomes (see Backus and Driffill (1985); Barro and Gordon (1983)). Romer and Romer (2000) provide empirical evidence that the Fed does have inflation information superior to that known by commercial forecasters.

Clearly, the monetary literature has recognized the role of information diffusion, from policymakers to its citizens, and how this influences macroeconomic stabilization policy.[2] Yet, there is currently no study analyzing how interactions among citizens affect the stabilizing outcome of monetary policy. The information diffusion among different groups of citizens is perhaps a most natural process because people do not interpret the public information in an identical manner (see Kandel and Zilberfarb (1999)).

The purpose of this paper is to provide a framework, in which we can assess a dynamic information diffusion process among different groups of agents (in a self-referential model). We will build our framework within a general cobweb-type expectation model because of its wide-applicability and popularity among macroeconomic studies (see Ezekiel (1938); Muth (1961); Arifovic (1994); Brock and Hommes (1997); Evans and Honkapohja (2001); and Branch (2002)). Lucas (1973), in particular, models inflation expectations with a cobweb model.

Our version of the cobweb model will allow for both information heterogeneity and information diffusion. In the inflation forecasting literature

[1] This line of research is largely inspired by conversations with many influential market participants who emphasize that their decisions are highly influenced by other market participants.

[2] Carlson and Valev (2002) show in theory that the proportion of agents who form rational expectations can affect the speed and the overall performance of a disinflation policy. They argue that a central bank would have an incentive to choose how much information to disseminate since this choice can affect the number of informed agents in the economy.

recent empirical studies demonstrate that information heterogeneity and diffusion exist. Carlson and Valev (2001) present evidence that a currency board, used to strengthen the credibility of a central bank's disinflation announcement, lowers the inflation expectations/forecasts less for agents with less educational level than for agents with a higher education level. In a related investigation, also add a further wrinkle. Information diffusion is asymmetric flowing from the more informed to the less informed. Granato and Krause (2000) show that the inflation forecasts of a better educated group influence the less educated group's forecasts, but not vice versa.

Both studies share the view that educational differences contribute to the asymmetry. Their results constitute two fundamental assumptions for our model: (1) agents with less (more) education formed less- (more-) informed expectations and (2) agents with less education rely on the forecasts from agents with more education to make their own forecasts. Accordingly, in our framework we include two groups of agents with uneven information levels due to educational differences. A group with higher (lower) education level is considered to have the ability to form more- (less-) informed expectations. To improve forecasting precision, the less-informed group of agents will try to acquire useful information from the more-informed group by observing their forecasting behavior. Yet, with less ability to interpret the content of information, observational errors due to misinterpretation of others' actions would naturally occur in the information acquisition process.

Using this setup, we study the information interaction process of how the more-informed group's forecasts are influenced by the forecasts of the less-informed group. We find the "boomerang effect", a situation in which the inaccurate forecasts of a less-informed group confound a more-informed group's forecasts when they try to acquire information from more-informed group but with mistakes. In other words, even when one group of agents has full information to make forecasts, they cannot obtain the rational expectation equilibrium (REE) due to the interaction with another group with less accurate forecasting abilities. Not only is the REE unobtainable, but the forecast errors of these well-informed agents become larger.

Employing quarterly survey data (1978:I to 2000:II) on inflation expectations obtained from the Survey Research Center (SRC) at the University of Michigan, we examine if the boomerang effect exists among agents for different educational groups.[3] Our data indicate that all the variables in

[3]See Carlson and Valev (2001) and Granato and Krause (2000) for examples of using

the model contain unit roots and exhibit potential long run serial (inter-) dependencies. Applying cointegration and Granger causality tests, we find robust evidence for the existence of a boomerang effect in agents' inflation forecasting behavior.

The paper is organized as follows. Section 2 introduces a standard cobweb-type framework, which we use to study the interaction between more and less-informed groups of agents. In section 3, we present and discuss the boomerang effect. Section 4 briefly outlines the testable hypotheses from our theoretical model. We then report the supporting evidence for the existence of a boomerang effect using surveyed inflation expectations data. Section 5 concludes.

2. Theoretical Framework

The original cobweb model introduced in Ezekiel (1938) and Muth (1961) is a reduced form of supply and demand in an isolated market. It represents a single competitive market where a time lag exists in production. The market equilibrium of price and quantity is determined by firms' expected profit maximization and with demand determined exogenously.[4] In recent years, both macroeconomists and financial economists revise the original cobweb model to suit various interests. Stein (1992) applies the cobweb model to study rational expectations in forward and futures markets. Arifovic (1994), Brock and Hommes (1997), and Branch (2002) use cobweb models to examine learning dynamics. Wenzelburger (2002) generalizes a cobweb monetary framework to explore an adaptive learning scheme for an overlapping generation model with pure exchange.

We apply the cobweb model to the investigation of economic forecasting behavior within a dynamic information diffusion process. In theory, our approach is consistent with Lucas (1973), and Evans and Honkapohja (2001), where they both confine the attention on the "inflation" forecasting/expectation behavior within a cobweb-type model. Moreover, the theoretical findings from our paper would have wider-usage for future research because they are applicable to the discussion of any economic forecasting behavior other than just inflation.

We assume the economy operates under the cobweb-type expectations model which represents the data generation process (DGP) or true model:

education level to categorize survey respondents.

[4]We detail the complete description of a standard cobweb model in the Appendix.

$$y_t = \alpha + \beta E^*_{t-1} y_t + \gamma' w_{t-1} + \eta_t, \tag{1}$$

where y_t is an endogenous variable which can be thought as any economic variable that agents are interested in forecasting on, $E^*_{t-1} y_t$ is the (rational or nonrational) expectation of y_t formed at the end of time $t-1$, w_{t-1} is a $m \times 1$ vector of exogenous observables following a stationary VAR (without loss of generality, we assume w_t has zero mean), γ' is a $1 \times m$ vector, and η_t is $iid(0, \sigma^2_\eta)$.

Our model contains two groups of agents. We define Group L as the less-educated group. These agents tend to be less current on economic events or institutional changes and thereby they are less-informed. We characterize members of the second group, Group H, as opinion leaders who are better educated and up-to-date on economic events and institutional changes and thereby they are more-informed. These opinion leaders are key in any information diffusion process since they are recognized by the less educated group to have superior information. We distribute these two groups evenly in the population.

Agents are assumed to act like econometricians and forecast y_t by running least squares regressions of y_t on past information w_{t-1}.[5] If agents forecast independently with the same observable information, then the forecast (or perceived law of motion, (PLM)) for each group is given as:[6]

$$y_t = a_{i,t-1} + b'_{i,t-1} w_{t-1} + v_t, \tag{2}$$

where $i \in \{L, H\}$, and $E(v_t) = 0$. Subscripts L and H represent Group L and Group H, respectively.

Two groups are assumed to have different information sets (x_{t-1},w_{t-1}). The Group H has the information set of w_{t-1}, while the Group L has the information set of x_{t-1}. For simplicity, w_t is a 2×1 vector of exogenous observations and x_t is a 1×1 vector of exogenous observations where $x_t \subset w_t$. With the empricial support by Granato and Krause (2000), we further assume that agents are interactive, and that Group L observes Group H's expectations to make its forecast (but not vice versa).[7] Therefore, Group L's

[5]See Evans and Honkapohja (2001) for the basic background on adaptive learning.

[6]See Evans and Honkapohja (2001) for the case in which the two groups learn the REE independently.

[7]Bomfim (2001) suggests an opposite assumption. He uses a dynamic real business cycle model in which there are sophisticated or rule-of-thumb agents in an economy. He

PLM is:

$$y_t = a_{L,t-1} + b_{L,t-1}x_{t-1} + c_{L,t-1}\widehat{y}_{t-1} + v_t \tag{3}$$

and

$$\widehat{y}_{t-1} = E^*_{H,t-1}y_t + \tilde{e}_{L,t-1}, \tag{4}$$

where $\tilde{e}_{L,t-1} \sim iid\left(0, \sigma^2_{\tilde{e}_L}\right)$ and $\widehat{y}_{t-1}$ is the observed information that Group L gets from Group H, $E^*_{H,t-1}y_t$ (see equation (6)) with observational error $(\tilde{e}_{L,t-1})$ at time $t-1$. Since Group L obtains the observed information after Group H forms its expectations, Group L treats the observed information as a predetermined variable.

As noted earlier in the introduction the less-informed agents would acquire information from more-informed agents. This borrowed information includes the expectations from more-informed agents. However, less-informed agents could experience some difficulty in understanding these expectations, and they may interpret the more-informed agents' information differently themselves.[8] It is also intuitively reasonable to believe agents are not able to obtain the exact information from others. Therefore, we impose a distribution of observational errors, $\tilde{e}_{L,t-1}$, to indicate the degree of misinterpretation of others' actions.

The highly informed group, Group H, possesses the full information to forecast the variable of interest. Since $x_{t-1} \subset w_{t-1}$, we can partition the information set w_{t-1} into two parts: $w_{t-1} \equiv \begin{pmatrix} x_{t-1} \\ w_{2,t-1} \end{pmatrix}$ and $b'_{H,t-1} \equiv \begin{pmatrix} b_{1H,t-1} \\ b_{2H,t-1} \end{pmatrix}$, where $w_{2,t}$ is a 1×1 vector of exogenous observations. We call this vector advanced information. To simplify the analytical calculations we assume that the covariance between x_t and $w_{2,t}$ is zero $(cov\left(x_t, w_{2,t}\right) = 0)$.[9] Therefore, the PLM for Group H is:

$$y_t = a_{H,t-1} + b_{1H,t-1}x_{t-1} + b_{2H,t-1}w_{2,t-1} + v_t. \tag{5}$$

assumes that the sophisticated agents form their expectations by forecasting the decisions of the less sophisticated rule-of-thumb agents. His results indicate that the aggregate properties of the economy are influenced by the rule-of-thumb agents.

[8] Kandel and Zilberfarb (1999) argue that people do not interpret the existing information in an identical way. Using Israeli inflation forecast data, they show that the hypothesis of identical-information interpretation is rejected.

[9] The numerical results in the case that $cov\left(x_t, w_{2,t}\right) \neq 0$ are also considered. The results can be obtained on request.

We assume that Group L and Group H forecast following the process of equations (3) and (5), respectively, and that they have data on the political economic system from periods $t_i = T_i, ..., t-1$, where $i \in \{L, H\}$. Therefore, the time $t-1$ information set for the less-informed group, Group L, is $\{y_i, x_i, \widehat{y}_i\}_{i=T_L}^{t-1}$ but the information set for Group H at time $t-1$ is $\{y_i, w_i\}_{i=T_H}^{t-1}$. The two groups use (6) to forecast the variable of interest (inflation rate):[10]

$$E^*_{i,t-1} y_t = \varphi'_{i,t-1} z_{i,t-1}, \tag{6}$$

where $i \in \{L, H\}$, $z'_{L,t-1} \equiv (1, x_{t-1}, \widehat{y}_{t-1})$, $z'_{H,t-1} \equiv (1, x_{t-1}, w_{2,t-1})$, $\varphi'_{L,t-1} \equiv (a_{L,t-1}, b_{L,t-1}, c_{L,t-1})$ and $\varphi'_{H,t-1} \equiv (a_{H,t-1}, b_{1H,t-1}, b_{2H,t-1})$.

Since we assume that both groups are evenly distributed, equation (1) shows that the current situation of the economy, y_t, depends on the average expectations from both groups, $E^*_{t-1} y_t$.

3. The Boomerang Effect

3.1. Restrictive Perceptions Equilibrium

In this case, one group has a restricted information set, but this does not preclude the possibility for optimal forecasting. This situation describes the results in a restricted perceptions equilibrium (RPE), which Evans and Honkapohja (2001) define as an optimal forecast in the face of a restricted information set (used by agents) (See also Marcet and Sargent (1989b), and Sargent (1991)).

Using a stochastic recursive algorithm, we can solve for the RPE as follows:

$$\bar{\varphi}_L = \begin{pmatrix} \bar{a}_L \\ \bar{b}_L \\ \bar{c}_L \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta}(1-\bar{c}_L) \\ \frac{\gamma_1}{1-\beta}(1-\bar{c}_L) \\ \frac{(2\gamma_2+\beta\bar{b}_{2H})\bar{b}_{2H}\sigma^2_{w2}}{(2-\beta)(\sigma^2_{\tilde{e}_L}+\bar{b}^2_{2H}\sigma^2_{w2})} \end{pmatrix} \tag{7}$$

and

$$\bar{\varphi}_H = \begin{pmatrix} \bar{a}_H \\ \bar{b}_{1H} \\ \bar{b}_{2H} \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{1-\beta} \\ \frac{\gamma_1}{1-\beta} \\ \frac{2\gamma_2}{2-\beta(1+\bar{c}_L)} \end{pmatrix}, \tag{8}$$

where $\gamma \equiv (\gamma_1, \gamma_2)$.

The observational error $\tilde{e}_{L,t-1}$ plays a very important role in the model. Whether Group L uses the observed information from Group H depends on

[10] We note that since the observed information $\widehat{y}_{t-1}$ is predetermined, the expectation operator in equation (6) has no affect on $\widehat{y}_{t-1}$at time $t-1$.

how accurately the less-informed group understands information (the expectations) from the highly informed group.

The accuracy is represented by the variance of the observational error, $\sigma^2_{\tilde{e}_L}$. Equation (7) implies that $0 \leq \bar{c}_L \leq 1$. If Group L can fully understand and make use of Group H's expectations (i.e., $\sigma^2_{\tilde{e}_L} \to 0$), then we can see that $\bar{c}_L = 1$ (by solving equations (7) and (8) with $\sigma^2_{\tilde{e}_L} = 0$.). In addition, $\bar{c}_L \to 0$ as $\sigma^2_{\tilde{e}_L} \to \infty$. Moreover, the values of $\bar{c}_L$ affect $\bar{a}_L$ and $\bar{b}_L$. If $\bar{c}_L \to 0$, $\bar{a}_L \to \frac{\alpha}{1-\beta}$ and $\bar{b}_L \to \frac{\gamma_1}{1-\beta}$, and both $\bar{a}_L$, $\bar{b}_L \to 0$ if $\bar{c}_L \to 1$.

However, under the assumption that the covariance between x_t and $w_{2,t}$ is zero, $\bar{c}_L$ does not affect $\bar{a}_H$ and $\bar{b}_{1H}$ at all. Both will approach the REE, $\left(\bar{a}_H, \bar{b}_{1H}\right) \to \left(\frac{\alpha}{1-\beta}, \frac{\gamma_1}{1-\beta}\right)$.[11] Equation (8) shows that $\bar{b}_{2H}$ is affected by $\bar{c}_L$, ranging between $\left(\frac{\gamma_2}{1-\beta}, \frac{2\gamma_2}{2-\beta}\right)$. This latter relation is evidence of what we call *the boomerang effect on the RPE*: the observational error of the less-informed group biases the parameter(s) of the highly informed group's forecasting rule.

3.2. Forecast Accuracy: Mean Square Errors

It is clear that Group L places some weight on the observed information from Group H. Group L uses Group H's expectations (i.e., higher $\bar{c}_L$) as long as Group L does not have a large variation in observation error in understanding Group H's information (i.e., lower $\sigma^2_{\tilde{e}_L}$). We now consider how both groups' forecast accuracy is affected if the less-informed group uses the observed expectations from the highly informed group. To show this, we calculate the mean squared error (MSE) for the forecasts of Groups L and H, respectively:[12]

$$MSE_L = \begin{cases} \sigma^2_\eta & \text{if} \quad \sigma^2_{\tilde{e}_L} = 0 \\ \left[\frac{2\gamma_2(1-\bar{c}_L)}{2-\beta(1+\bar{c}_L)}\right]^2 \sigma^2_{w_2} + \left[\frac{(\beta-2)\bar{c}_L}{2}\right]^2 \sigma^2_{\tilde{e}_L} + \sigma^2_\eta & \text{if} \quad \sigma^2_{\tilde{e}_L} \in (0,\infty) \\ \left(\frac{2\gamma_2}{2-\beta}\right)^2 \sigma^2_{w_2} + \sigma^2_\eta & \text{if} \quad \sigma^2_{\tilde{e}_L} \to \infty \end{cases} \quad (9)$$

$$MSE_H = \begin{cases} \sigma^2_\eta & \text{if} \quad \sigma^2_{\tilde{e}_L} = 0 \text{ or } \sigma^2_{\tilde{e}_L} \to \infty \\ \left(\frac{\beta\bar{c}_L}{2}\right)^2 \sigma^2_{\tilde{e}_L} + \sigma^2_\eta & \text{if} \quad \sigma^2_{\tilde{e}_L} \in (0,\infty) \end{cases}, \quad (10)$$

where $MSE_i \equiv MSE\left(y^*_{i,t+1|t}\right)$ for $i \in \{L, H\}$.

[11] If the $cov\left(x_t, w_{2,t}\right) \neq 0$, then the numerical simulations (not present here) show $\bar{b}_{1H}$ is also affected by the less informed group's forecast errors.

[12] For comparison, we also calculate the MSEs for situations in which both groups have the same (full) information set and learn independently. We show that both groups' MSE's are at a minimum when $MSE_L = MSE_H = \sigma^2_\eta$.

Equation (9), which uses different values of $\sigma^2_{\tilde{e}_L}$, depicts the accuracy of the less-informed group's predictions. According to the first expression of equation (9), where $\sigma^2_{\tilde{e}_L} = 0$, Group L is able to fully understand the expectations from Group H (i.e., without any observation errors). The result is that Group L obtains the minimum MSE ($MSE_L = \sigma^2_\eta$).

However, omitted variables and observational error problems reduce the Group L's predictive accuracy. This outcome can be seen in the second and third conditions of equation (9). The variances of the omitted variable $\left(\sigma^2_{w_2}\right)$ and observation errors $\left(\sigma^2_{\tilde{e}_L}\right)$ appear in those conditions and make the MSE larger than σ^2_η.

The results for Group H indicate that only the two limit points of the variance of the observation errors $\left(\sigma^2_{\tilde{e}_L} = 0 \text{ or } \sigma^2_{\tilde{e}_L} \to \infty\right)$ produce the most efficient outcome. When $\sigma^2_{\tilde{e}_L} = 0$, in the limit, Group L uses the expectations from the highly informed group. Group L's expectations become exactly the same as those of Group H. Both groups can forecast efficiently. However, if $\sigma^2_{\tilde{e}_L} \to \infty$, Group L is not able to interpret Group H's expectations at all and eventually discards them. Both groups learn independently. Thus, the boomerang effect does not occur.

In the case of finite observation errors $\left(\sigma^2_{\tilde{e}_L} \in (0, \infty)\right)$, Group L observes and uses the expectations from Group H to form its prediction, which impairs the predictive accuracy of Group H's forecasts as follows: Our model (equation (1)) is self-referential, which means the expectations of y_t — $(E^*_{t-1}y_t)$ — of all agents affect the actual y at time t. The presence of observation error leads the less-informed group to form an incorrect expectation via the variable $\hat{y}_{t-1}$ in equation (6). This relation alters the actual y_t in equation (1), so when the highly informed group forms its new expectations of y_{t+1} at time t, using $\varphi_{H,t}$, there are errors and Group H fails to forecast the actual y_{t+1} correctly. Group H's predictions become less efficient, reflected by larger mean squared errors. This results in a boomerang effect on the MSE.

4. Empirical Analysis and Results

Equation (1) is commonly used in macroeconomic studies, especially in inflation models (Lucas (1973); Lucas and Sargent (1988); Evans and Honkapohja (2001)), and it serves as a good starting point for testing the boomerang effect. We use inflation expectations survey data from the SRC at the University of Michigan to identify whether the boomerang effect exists in inflation forecasting behavior.

For the boomerang effect to occur, two pieces of evidence must be present in the results. First, the information diffusion process must be asymmetric, starting from the more-informed (educated) agents to the less-informed (educated) agents. This evidence is considered to be the precondition for the occurrence of the boomerang effect. Second, the larger the observation error made by Group L agents in obtaining Group H agents' inflation forecasts, then greater the inaccuracy in inflation predictions by Group H agents.

4.1. Data: The Inflation Expectations Survey from SRC

Inflation expectations surveys are conducted by the SRC at the University of Michigan and the results are published in the Survey of Consumer Attitudes. Since 1978 the center has conducted monthly telephone interviews from a sample of at least 500 households randomly selected to represent all American households, excluding those in Alaska and Hawaii. Each monthly sample is drawn as an independent cross-section sample of households. Respondents selected in the drawing are interviewed once and then reinterviewed six months later. This rotating process creates a total sample made up of 60% new respondents and 40% prior respondents.

Survey respondents are asked approximately 50 core questions that cover three broad areas of consumer opinions: personal finances, business conditions, and buying conditions. In this paper, we consider the following questions that relate to measuring inflation expectations:

1. During the next 12 months, do you think that prices in general will go up, or go down, or stay where they are now?

2. By about what percent do you expect prices to go (up/down), on the average, during the next 12 months?

If respondents expect the price level will go up (or down) on question 1, they are asked in the second question to provide the exact percent the price level will increase (or decrease), otherwise the second question is coded as zero percent.[13]

We divide our inflation expectation survey data into different educational categories. To be consistent with our theory, we consider the respondents with college or graduate degrees as the highly informed group (Group H) and those without college degree as the less-informed group (Group L). Based on

[13] Some respondents may confuse the concepts of "change in level" and "change in rate." To account for this confusion, respondents who answer "stay the same" on question 1 are asked the following to eliminate any confusion: "Do you mean that prices will go up at the same rate as now, or that prices in general will not go up during the next 12 months?" For more details on the survey procedures, see http://www.sca.isr.umich.edu/.

the unique characteristics of our data set, we are able to separate Group L into two distinct levels: (1) high school diploma or some college (denoted as "$L1$"); and (2) less than high school or no high school diploma (denoted as "$L2$").

Since the survey sample is randomly selected every month, the number of respondents in different educational categories can also be different. These variations can create insufficient monthly data for the same number of education levels. To minimize this problem we convert the monthly data into quarterly data covering the period of 1978:I to 2000:II.

4.2. Testing the Precondition for the Boomerang Effect: Asymmetric Information Diffusion

Panels A and B of Figure 1 graph inflation forecasting data and inflation forecast errors, respectively. The groupings are noted for the highly-educated group and two alternative less-educated groups. We note that the inflation forecasts and forecast errors share similar movements in the long run, although Group H expects lower inflation and also are more accurate than the less-educated groups.

[**INSERT FIGURE 1**]

Following Granato and Krause (2000), we examine the direction of information diffusion by means of Granger causality tests in a vector autoregression (VAR). Since there is evidence that the data possess a unit root, we use first differences for all classes of inflation forecasts. The Akaike information criterion (AIC) and Lagrange multiplier (LM) test statistics suggest the VAR system with lag order of seven is preferable on the basis of a minimum AIC with no serial correlation or heteroskedasticity in residuals.[14]

Table 1 reports results of the Granger causality tests. The null hypothesis that Group H does not Granger-cause Group L2 is rejected (p-value equals 0.030). However, Group H does not Granger-cause Group L1 (p-value equals 0.122). We also note that Group L1 Granger causes Group L2 (p-value equals 0.047). In contrast, Groups L1 and L2 do not Granger-cause Group H. We also find that Group L2 does not Granger-cause Group L1's forecasts. Overall, the testing results in Table 1 clearly indicate that

[14] Our unit root test results are based on both the augmented Dickey-Fuller test (1979), and the Elliott-Rothenberg-Stock test (1996). The results of the unit root tests and of the lag order selection for the VAR are available from the authors on request.

there is an asymmetric information diffusion: the inflation forecasts of the more-educated group(s) affect the less-educated groups.

Using these findings, Figure 2 depicts the dynamic responses of inflation forecasts from a less educated group from an innovation in a more educated group's inflation expectations. The results show that the less-educated agents do mimic agents with higher education level in a "positive" manner in revising their inflation expectations. The result supports the precondition of the boomerang effect.

[INSERT TABLE 1]
[INSERT FIGURE 2]

4.3. Testing The Boomerang Effect: Methodology

The central theoretical finding for the boomerang effect is that the larger observation errors made by the less-informed (less-educated) agents from obtaining more-informed (more-educated) agents' forecasts can eventually lead to less accurate forecasts by the more-informed groups of agents. Thus, the empirical evidence on the "positive" relation between the size of observation errors of less-informed agents and the size of forecasting inaccuracy of more-informed agents supports the existence of the boomerang effect. We measure the size of observation error $(\tilde{e}_{L,t})$ by its variance $\left(\sigma^2_{\tilde{e}_L}\right)$, and the size of forecasting inaccuracy by the mean square error of forecasts (MSE_H).

Consistent with the work of Sargent (1993), we assume that agents are boundedly rational and thus do not have full knowledge of the economy when forming expectations. Instead, agents use the least squares learning method to achieve REE in the long run. To reach the REE, the less-informed groups of agents repeatedly assess the following regression from equations (3) and (4):

$$E^*_{Lj,t-1}y_t = a_{Lj,t-1} + b_{Lj,t-1}x_{t-1} + c_{Lj,t-1}\left(E^*_{H,t-1}y_t + \tilde{e}_{Lj,t-1}\right) \qquad (11)$$

where $E^*_{Lj,t-1}y_t$ and $E^*_{H,t-1}y_t$ represent the inflation forecasts of less and more-informed groups, respectively, $j \in \{1,2\}$ and x_t is the information set for inflation forecasts.

The set of information includes the current and lagged federal funds rate, the current inflation rate, and oil prices.[15] We use twelve-year intervals to

[15] The data are from the FRED database provided by the Federal Reserve Bank of St. Louis.

represent the long-run period for agents to reach the REE. We construct the series, $\sigma^2_{\tilde{e}_{Lj}}$, using a rolling regression technique in which we fix the regression window of (11) at 49 quarters and move it forward every quarter.[16]

The observation error generated from equation (11) for less-informed groups is:

$$\tilde{e}_{Lj,t-1} = \frac{E^*_{Lj,t-1}y_t - a_{Lj,t-1} - b_{Lj,t-1}x_{t-1} - c_{Lj,t-1}E^*_{H,t-1}y_t}{c_{Lj,t-1}}.$$

This result follows that the variances of the observation error $\left(\sigma^2_{\tilde{e}_{Lj},t}\right)$ for the less-informed groups are:

$$\sigma^2_{\tilde{e}_{Lj},t} = \frac{\sum_t^{t+s} \tilde{e}^2_{Lj,t}}{s-1}, \ \forall t$$

where $s = 49$ represents the number of quarters in rolling windows.

We adopt the same rolling regression technique to estimate the mean square error for Group H:

$$MSE_{H,t} = \frac{\sum_t^{t+s}\left(y_t - E^*_{H,t-1}y_t\right)^2}{s}, \forall t$$

Our primary concern is the long-run (inter-)relation between MSE_H and $\sigma^2_{\tilde{e},Lj}$. We are interested in whether a larger value of $\sigma^2_{\tilde{e},Lj}$ causes MSE_H to increase. This result would be supportive evidence for the existence of the boomerang effect. To obtain consistent estimates of the unknown parameters entering the system consisting of MSE_H, $\sigma^2_{\tilde{e},L1}$,and $\sigma^2_{\tilde{e},L2}$, we first characterize the stochastic properties of these underlying variables.

Table 2 presents the augmented Dickey-Fuller (1979) and Elliott-Rothenberg-Stock (1996) test results. We find that MSE_H, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$ all contain a unit root. With test results indicating that we have all non-stationary variables in the system, the cointegration methodology is useful for exploring the long-run (inter-)relation among the variables. We use the Johansen test for this particular task.

[16] We also use 10-year and 15-year rolling regression windows in the empirical analysis. However, results from using different choices of regression windows do not show any substantive or statistical difference, indicating the robustness of empirical findings presented in the paper. Results based on 10- and 15-year rolling regression windows are available on request.

4.4. Empirical Results

Panel A in Table 3 reports the results of the cointegration tests of the long-run relation between MSE_H, and $\sigma^2_{\tilde{e},Lj}$. Columns 1 and 2 in Panel A summarize the results of cointegrating relations for two pairs of variables, $\left(MSE_H, \sigma^2_{\tilde{e},L1}\right)$ and $\left(MSE_H, \sigma^2_{\tilde{e},L2}\right)$, respectively. Both the maximum eigenvalues and trace statistics indicate that there are long-run equilibrium relations for both. Using the Johansen cointegration procedure, we find the cointegrating vectors of $\left(MSE_H, \sigma^2_{\tilde{e},Lj}\right)$ are $(1, -29.58)$ and $(1, -21.54)$ for $j \in \{1, 2\}$, respectively.

These results show a positive long-run equilibrium relation with the existence of the boomerang effect between MSE_H and $\sigma^2_{\tilde{e},L}$. It suggests that the mean square error on inflation forecasts for the respondents who hold a college degree or above (MSE_H) is positively related with the measurement errors resulting from the non-degree-holding respondents $\left(\sigma^2_{\tilde{e}_L}\right)$.

The results in column (3), where the cointegrating system consists of all three variables of MSE_H, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$, provides further evidence to support the boomerang effect found in columns (1) and (2). With an estimated cointegrating vector of $(MSE_H,\ \sigma^2_{\tilde{e},L1},\ \sigma^2_{\tilde{e},L2}) = (1, -20.52, -0.79)$, this robustness check shows that both $\sigma^2_{\tilde{e},L1}$ and $\sigma^2_{\tilde{e},L2}$ are positively related with MSE_H in the long run. The results in column (4) also show that the robust cointegrating vector among the three variables is solely the result of the boomerang effect since the variances of the measurement errors in the two levels of Group L are not cointegrated.

Furthermore, we examine if the boomerang effect is robust when both levels of Group L are combined. We examine this case by averaging the inflation expectations from Groups $L1$ and $L2$ to obtain $\sigma^2_{\tilde{e}_L}$. The cointegration estimation (not shown here) indicates that the boomerang effect is still robust where $\sigma^2_{\tilde{e}_L}$ is positively related with MSE_H in the long run.

Additional support for a boomerang effect occurs if we see that the direction of causality runs from $\sigma^2_{\tilde{e},L}$ to MSE_H (but not vice versa). Panel B of Table 3 gives the results of the Granger-causality tests using a vector error correction model (VECM). The results from systems (1) and (2) indicate that we can reject the null hypotheses that $\sigma^2_{\tilde{e},Lj}$ does not Granger causes MSE_H, for $j \in \{1, 2\}$. The respective test statistics are equal to 14.36 and 19.43 and are significant at the 0.05 level. On the other hand, we cannot reject the null hypothesis for reverse causation. Column (3) in Panel B report associated results which are highly consistent with findings in columns (1) and (2).

[INSERT TABLE 2]
[INSERT TABLE 3]

5. Conclusion

In this paper, we use a general cobweb expectations model to explore the forecasting behavior within an information diffusion process where a less-informed group interacts with a more-informed group. We focus particular attention on the boomerang effect, which we define as a situation in which the inaccurate forecasts of a less-informed group confound a more-informed group's forecasts.

We use the Survey Research Center (SRC) inflation expectations data to test the existence of the boomerang effect. The quarterly survey data, divided along different educational groups, covers 1978 to 2000. To test for the existence of the boomerang effect, we use the cointegration test to estimate the long run relationship between the variance of observational errors from the less educated group and the mean square error of the highly educated group's expectations. We find that the mean square error on inflation forecasts for the respondents who have higher education have a long-run positive relation to the observation errors resulting from the less educated respondents. In particular, the Granger causality test shows that this positive relation is only from the observation errors to the forecast accuracy. These results support the boomerang effect in the inflation forecasting behavior.

This paper also has implications for the overall performance of an inflation-stabilizing monetary policy. We have argued that if a segment of the public has less understanding of economic events, information diffusion creates a boomerang effect. Since the equilibrium forecasts in an economy are aggregations of agents' forecasts, a large boomerang effect can cause policymakers themselves to make inaccurate forecasts of economic conditions. The inaccurate forecasts can eventually cause additional economic volatility and failed stabilization policies.[17] To alleviate the boomerang effect, one normative policy suggestion is that policymakers should be more transparent about policy information. Transparency will make it possible for the less-informed citizens to better understand how the policy will work and hence make more accurate use of others with more information.[18] With more pre-

[17] A similar implication is also suggested by Bomfim (2001). See Footnote 7.

[18] There is research supporting this common-sense suggestion. Bernanke et al. (1999) notes that when information about the plans, objectives, or decisions of the monetary authorities are carefully explained, the public can more easily understand the contents of

cision in information acquisition, the less-informed citizens will confound a more-informed group's forecasts less and it can reduce the boomerang effect, improve policy effectiveness, and help with overall economic performance.

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a policy announcement.

Appendix. The Derivation of a Standard Cobweb Model

Arifovic (1994) and Evans and Honkapohja (2001) present a simple cobweb model in which there are n firms in a competitive market that produce a homogeneous product. Firms face a quadratic cost function of production:

$$c_{i,t} = f\tilde{q}^s_{i,t} + \frac{1}{2}gn\left(\tilde{q}^s_{i,t}\right)^2,$$

where $c_{i,t}$ represents firm i's production cost at $t-1$, $\tilde{q}^s_{i,t}$ is the planned production level, and $f \geq 0, g > 0$.

We assume that all firms face the exogenous market productivity shocks, $u^s_t = \lambda' w_{t-1} + v^s_t$, which are exclusive to their optimal planned production decisions. That is:

$$Q^s_t = \sum_{i=1}^{n} \tilde{q}^s_{i,t} + u^s_t,$$

where Q^s_t is the aggregate supply level, w_{t-1} is the $m \times 1$ vector of observable shocks at $t-1$, and v^s_t represents white noise unobservable shocks in productivity,[19] (i.e., $v^s_t \sim iid\left(0, \sigma^2_{v^s}\right)$).

Each individual firm i chooses the optimal planned individual quantity, $\tilde{q}^s_{i,t}$, to maximize its expected profit, $E^*_{t-1}\pi_{i,t}$, according to its (rational or nonrational) expectation of p_t formed at the end of time $t-1$, (i.e., $E^*_{t-1}p_t$):

$$\max_{\tilde{q}^s_{i,t}} E^*_{t-1}\pi_{i,t} = E^*_{t-1}\left[p_t q^s_{i,t} - f\tilde{q}^s_{i,t} - \frac{1}{2}gn\left(\tilde{q}^s_{i,t}\right)^2\right], \tag{12}$$

Equation (12) gives us the optimal planned production level for individual firm i:

$$\tilde{q}^s_{i,t} = (gn)^{-1}\left(E^*_{t-1}p_t - f\right),$$

Aggregate supply, $Q^s_t = \sum_{i=1}^{n} \tilde{q}^s_{i,t} + u^s_t$, is given as:[20]

$$Q^s_t = \chi_1 E^*_{t-1}p_t + \chi'_2 w_{t-1} + v^s_t,$$

where $\chi_1 = g^{-1} > 0$, $\chi'_2 \equiv \lambda'$.

[19] Branch and Evans (2003) only allow for exogenous unobservable productivity shocks. We generalize this situation by assuming that firms faces both observable and unobservable market productivity shocks, w_{t-1} and v^s_t, respectively. Both shocks are independent of each other (i.e., $E\left(w_{t-1}, v^s_t\right) = 0$). We further assume that w_{t-1} and v^s_t are independent of $\tilde{q}^s_{i,t}$.

[20] Without loss of generality, we assume $f = 0$.

The market price, p_t, which clears the market at time t is also determined by market demand:

$$Q_t^d = \vartheta_0 - \vartheta_1 p_t + \upsilon_t^d,$$

where ϑ_0 is an intercept, $\vartheta_1 > 0$, w_t is an $m \times 1$ vector of demand shocks, and υ_{dt} are white noise demand shocks.

In equilibrium ($Q_t^d = Q_t^s$), the reduced-form of the model is:

$$p_t = \alpha + \beta E_{t-1}^* p_t + \gamma' w_{t-1} + \eta_t, \tag{13}$$

where $\alpha \equiv \vartheta_0/\vartheta_1, \beta \equiv -\chi_1/\vartheta_1 < 0$, $\gamma' = -\chi_2'/\vartheta_1$, $\eta_t = \left(\upsilon_t^d - \upsilon_t^s\right)/\vartheta_1$, and $\eta_t \sim iid\left(0, \sigma_\eta^2\right)$. In equation (13), the market price (p_t) is determined by its expectation $\left(E_{t-1}^* p_t\right)$ and other observable factors (w_{t-1}) and stochastic shocks (η_t).

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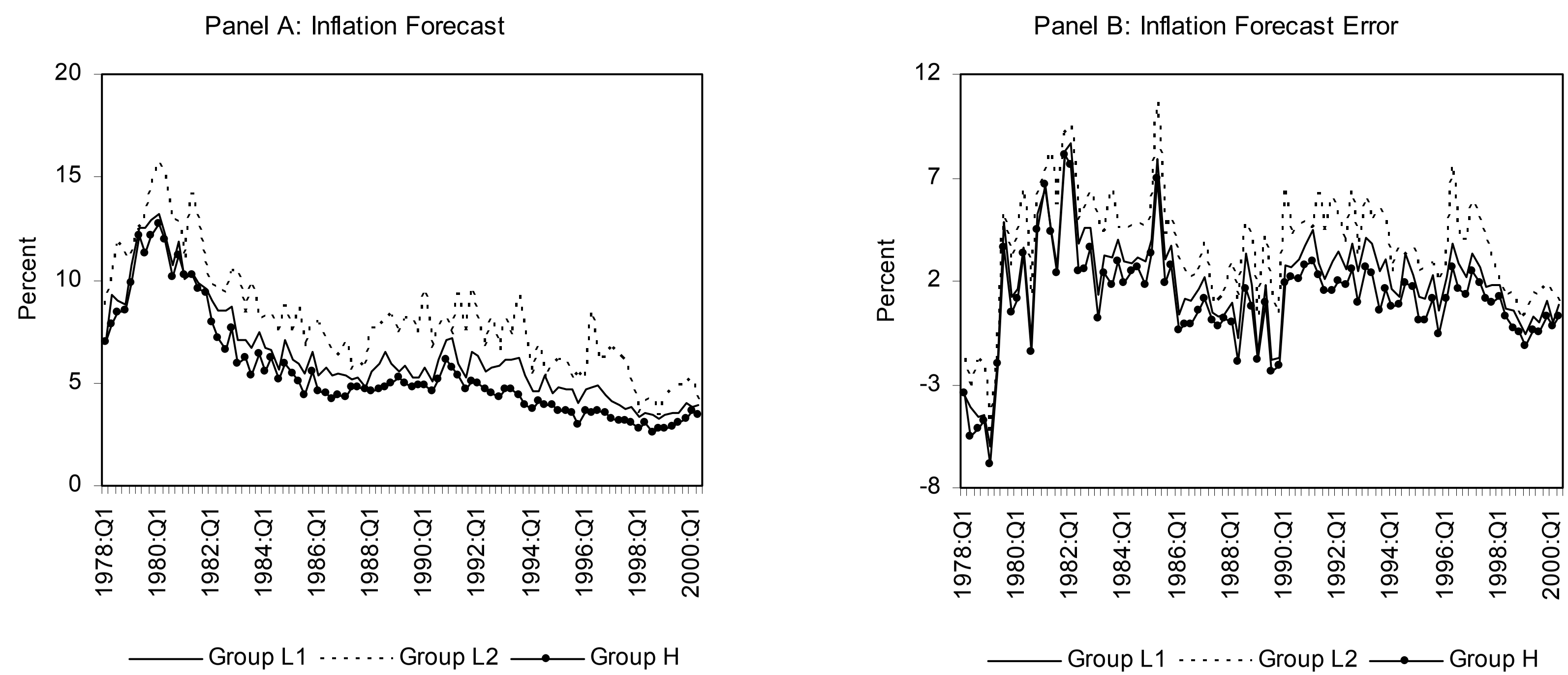

Fig. 1. Inflation forecasts and forecast errors for agents in three educational categories. Group L1 represents agents with a high school diploma or some college. Group L2 represents agents with less than or no high School diploma. Group H represents agents with a college degree or graduate degree. The data source is the SRC at the University of Michigan.

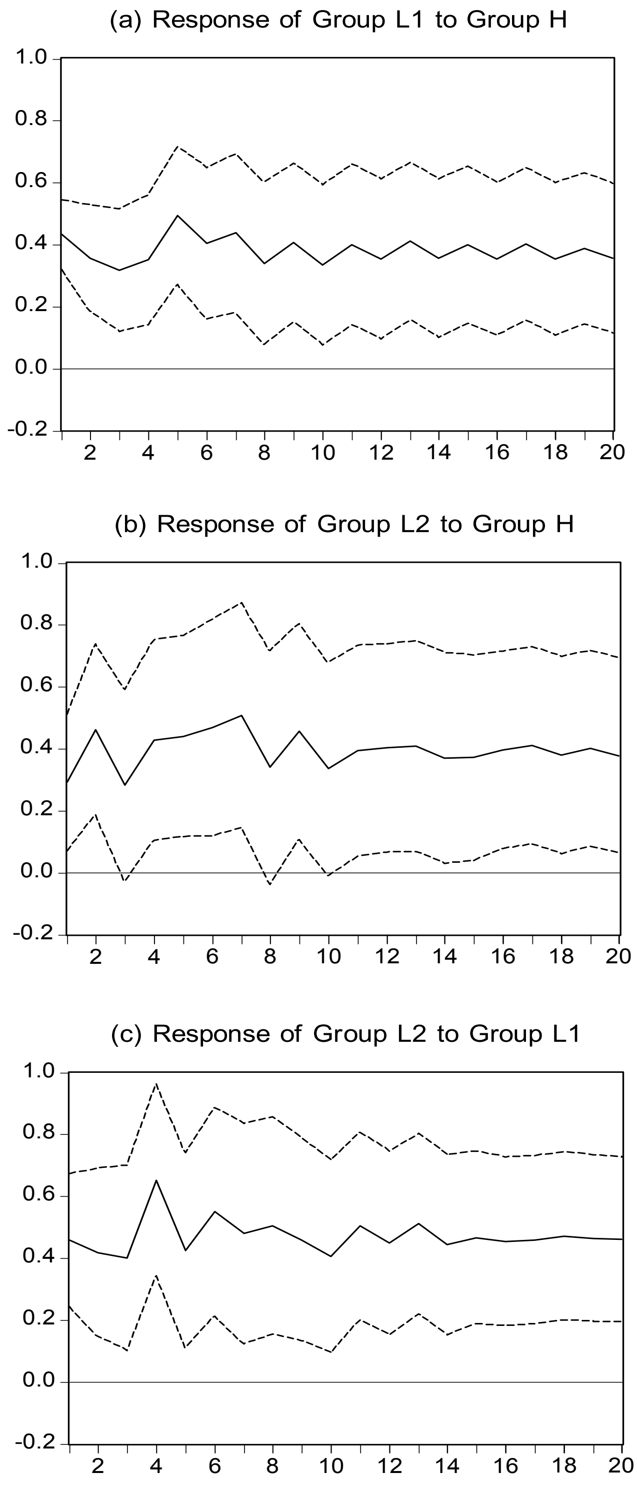

Fig. 2. Accumulated response to one S.D. innovations ± 2 standard errors. Dashed lines denote 95 percent confidence intervals. Group L1 represents agents with a high school diploma or some college. Group L2 represents agents with less than or no high school diploma. Group H represents agents with a college degree or graduate degree. The data source is the SRC at the University of Michigan.

Table 1

VAR pairwise Granger causality test: the direction of information diffusion across Group H, Group L1 and Group L2

If forecasts of the *higher* educated group Granger-cause those of the *less* educated group?

	Null Hypothesis	Chi-sq statistics [*P*-value]
a.	Group H does not Granger-cause Group L1	11.401 [0.122]
b.	Group H does not Granger-cause Group L2	15.522** [0.030]
c.	Group L1 does not Granger-cause Group L2	14.253** [0.047]

If forecasts of the *less* educated group Granger-cause those of the *higher* educated group?

	Null Hypothesis	Chi-sq statistics [*P*-value]
d.	Group L1 does not Granger-cause Group H	3.897 [0.792]
e.	Group L2 does not Granger-cause Group H	7.583 [0.371]
f.	Group L2 does not Granger-cause Group L1	2.603 [0.919]

** indicates statistical significance at 5 percent.

Table 2

Unit root test results: the integration properties of MSE_H, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$

A. Data in levels

	Augmented Dickey-Fuller Test		Elliott-Rothenberg-Stock Test			
Variable	DF_μ [a]	DF_τ [b]	Optimal Lag	$DF-GLS_\mu$ [c]	$DF-GLS_\tau$ [c]	Conclusion
MSE_H	-2.222	-0.661	3	-0.305	-1.690	I(1)
$\sigma^2_{\tilde{e},L1}$	-0.826	-2.797	3	-0.531	-2.638	I(1)
$\sigma^2_{\tilde{e},L2}$	-1.896	-3.327*	6	-0.743	-3.327**	I(1)

B. Data in first differences

Variable	DF_μ [a]	DF_τ [b]	Optimal Lag	$DF-GLS_\mu$ [c]	$DF-GLS_\tau$ [c]	Conclusion
MSE_H	-4.536***	-4.966***	2	-2.041*	-2.371**	I(0)
$\sigma^2_{\tilde{e},L1}$	-7.616***	-7.588***	2	-2.957***	-2.973*	I(0)
$\sigma^2_{\tilde{e},L2}$	-7.002***	-6.926***	7	-3.367***	-3.440**	I(0)

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively.

a. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -3.597 and -2.934, respectively.

b. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. Fuller (1976) 1 and 5 percent critical values for a sample size of 41 equal -4.196 and -3.522, respectively.

c. Test allows for a constant; one-sided test of the null hypothesis that the variable is nonstationary. The critical values, not reported here, are calculated from the response surface estimates of Table 1, Cheung and Lai (1995).

Table 3

Johansen cointegration tests and Granger causality tests in VECM: The (inter-)relation among MSE_H, $\sigma^2_{\tilde{e},L1}$, and $\sigma^2_{\tilde{e},L2}$

A: Rank test and cointegrating relation

	Variables in the System							
	$MSE_H, \sigma^2_{\tilde{e},L1}$ [a] (1)		$MSE_H, \sigma^2_{\tilde{e},L2}$ [b] (2)		$MSE_H, \sigma^2_{\tilde{e},L1}, \sigma^2_{\tilde{e},L2}$ [c] (3)		$\sigma^2_{\tilde{e},L1}, \sigma^2_{\tilde{e},L2}$ [d] (4)	
Null Hypothesis	$\hat{\lambda}_{\max}$	Trace	$\hat{\lambda}_{\max}$	Trace	$\hat{\lambda}_{\max}$	Trace	$\hat{\lambda}_{\max}$	Trace
No rank	12.82**	15.22**	8.00	12.20*	48.60***	87.52***	6.52	8.80
	[11.44]	[12.53]	[11.44]	[12.53]	[22.00]	[34.91]	[11.44]	[12.53]
At most 1 rank	2.40	2.40	4.20	4.20	32.65***	38.92***	2.28	2.28
	[3.84]	[3.84]	[3.84]	[3.84]	[15.67]	[19.96]	[3.84]	[3.84]
At most 2 ranks	-	-	-	-	6.27	6.27	-	-
					[9.24]	[9.24]		
Conclusion	1 cointegrating relation		1 cointegrating relation		2 cointegrating relations		None	
Estimated Cointegration Vector	$(MSE_{High}, \sigma^2_{\tilde{e},L1}) =$ (1, -29.58)		$(MSE_{High}, \sigma^2_{\tilde{e},L2}) =$ (1, -21.54)		$(MSE_{High}, \sigma^2_{\tilde{e},L1}, \sigma^2_{\tilde{e},L2}) =$ (1, -20.52, -0.79)		None	

B: The direction of causality in VECM

	Variables in the System			
	$MSE_H, \sigma^2_{\tilde{e},L1}$ [a] (1)	$MSE_H, \sigma^2_{\tilde{e},L2}$ [b] (2)	$MSE_H, \sigma^2_{\tilde{e},L1}, \sigma^2_{\tilde{e},L2}$ [c] (3)	$\sigma^2_{\tilde{e},L1}, \sigma^2_{\tilde{e},L2}$ [d] (4)
Null Hypothesis	Chi-sq statistics [*P*-value]	Chi-sq statistics [*P*-value]	Chi-sq statistics [*P*-value]	Chi-sq statistics [*P*-value]
$\sigma^2_{\tilde{e},L1}$ does not cause MSE_H	14.36***	-	21.04***	-
	[0.006]		[0.007]	
MSE_H does not cause $\sigma^2_{\tilde{e},L1}$	3.82	-	5.68	-
	[0.430]		[0.682]	
$\sigma^2_{\tilde{e},L2}$ does not cause MSE_H	-	19.43***	30.87***	-
		[0.000]	[0.000]	
MSE_H does not cause $\sigma^2_{\tilde{e},L2}$	-	4.72	7.15	-
		[0.194]	[0.521]	

***, **, and * indicate statistical significance at 1, 5 and 10 percent, respectively. We use the AIC criterion to choose the optimal number of lags to be included in each empirical model. 5 percent critical values, from Osterwald-Lenum (1992), for rank tests are in parentheses.

a. Test allows for a constant but no trend in the data space and 4 lags are included in the system.

b. Test allows for a constant but no trend in the data space and 3 lags are included in the system.

c. Test allows for a constant but no trend in the cointegration space and 8 lags are included in the system.

d. Test allows for a constant but no trend in the data space and 4 lags are included in the system.